

Minisuperspace as a Quantum Open System

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Abstract

We trace the development of ideas on dissipative processes in chaotic cosmology and on minisuperspace quantum cosmology from the time Misner proposed them to current research. We show

- 1) how the effect of quantum processes like particle creation in the early universe can address the issues of the isotropy and homogeneity of the observed universe,
- 2) how viewing minisuperspace as a quantum open system can address the issue of the validity of such approximations customarily adopted in quantum cosmology, and
- 3) how invoking statistical processes like decoherence and correlation when considered together can help to establish a theory of quantum fields in curved spacetime as the semiclassical limit of quantum gravity.

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1. Introduction

In the five years between 1967 and 1972, Charlie Misner made an idelible mark in relativistic cosmology in three aspects.

First he introduced the idea of chaotic cosmology. In contrast to the reigning standard model of Friedmann-Lemaitre-Robertson-Walker universe where isotropy and homogeneity are ‘put in by hand’ from the beginning, chaotic cosmology assumes that the universe can have arbitrary irregularities initially. This is perhaps a more general and philosophically pleasing assumption. To reconcile an irregular early universe with the

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observed large scale smoothness of the present universe, one has to introduce physical mechanisms to dissipate away the anisotropies and inhomogeneities. This is why dissipative processes are essential to the implementation of the chaotic cosmology program. Misner (1968) was the first to try out this program in a Bianchi type-I universe with the neutrino viscosity at work in the lepton era. Though this specific process was found to be too weak to damp away the shear, the idea remains a very attractive one. As we will see, it can indeed be accomplished with a more powerful process, that of vacuum particle production at the Planck time. The philosophy of chaotic cosmology is similar to that behind the inflationary cosmology of Guth (1981), where the initial conditions are rendered insignificant by the evolutionary process, which, in this case is inflation.

Second, the chaotic cosmology program ushered in serious studies of the dynamics of Bianchi universes. This was exemplified by Misner's elegant work on the mixmaster universe (1969a). (That was when one of us first entered the scene.) The ingenious use of pictorial representation of the curvature potentials made the complicated dynamics of these models easy to follow and opened the way to a systematic analysis of this important class of cosmology. (See Ryan and Shepley 1975.) This development complements and furthers the ongoing program of Lifshitz, Khalatnikov and Belinsky (1963, 1970) where, in seeking the most general cosmological solutions of Einstein's equations using rigorous applied mathematics techniques (see also Eardley, Liang and Sachs 1971), they found the inhomogeneous mixmaster universe (the 'generalized' Kasner solution) as representing a generic behavior near the cosmological singularity. This also paralleled the work of Ellis and MacCallum (1969) who, following Taub (1951) and Heckmann and Schucking (1962), gave a detailed analysis of the group-theoretical structure of the Bianchi cosmologies. Misner's chaotic cosmology program provided the physical rationale for these studies. It also prepared the ground for minisuperspace quantum cosmology (1969b, 1972).

Third, the establishment of the quantum cosmology program, which includes the application of the ADM quantization (1962) to cosmological spacetimes, and the adaptation of the Wheeler- DeWitt superspace formulation of quantum gravity (Wheeler 1968, De Witt 1967) to minisuperspace cosmology. The physical motivation was to understand 'the issue of the initial state' (Wheeler 1964), and, in particular, the quantum effects of gravity on the cosmological singularity. It was Misner and his associates who started the first wave of activity in quantum cosmology (Ryan, 1972; see also Kuchar 1971, Berger 1974, 1975). The second wave, as we know, came with the work of Hartle and Hawking (1983), and Vilenkin (1983) who, while formulating the problem in Euclidean path integral terms, opened the question on the boundary conditions of the universe.

These three directions in cosmology initiated by Misner and his collaborators in the early seventies –chaotic cosmology, dissipative processes and minisuperspace quantum cosmology– have developed in major proportions in the past two decades. A lot of

current research work in these areas still carry the clear imprint of his influence. The present authors have had the good fortune to be influenced by his way of thinking. We want to trace out some major developments since Misner wrote these seminal papers and describe the current issues in these areas, with more emphasis on their interconnections from early universe quantum processes to quantum cosmology.

1.1. Particle Creation as a Quantum Dissipative Process

Misner (1968) and Matzner and Misner (1972) showed that neutrino viscosity in the lepton era ($\approx 1\text{sec}$ from the Big Bang) is in general not strong enough to dissipate away the shear in the Bianchi universes. It was Zel'dovich (1970) who first suggested that vacuum particle creation from the changing gravitational field in anisotropic spacetimes may act as a powerful dissipative source. This created a real possibility for the idea of chaotic cosmology to work, now relying on quantum field processes effective near the Planck time (10^{-43} sec from the Big Bang). One needs a new theory — the theory of quantum fields in curved spacetime. Actually this theory was just beginning to take shape through the work of Parker (1966, 1969) and Sexl and Urbantke (1968) at about the same time when Misner was working on the isotropy of the universe problem (1968). It was later realized that the reason why the Bianchi I universe and not the Robertson-Walker universe produces a copious amount of particles is because it breaks the conformal invariance of the theory (Parker 1973). Zel'dovich used a simple dimensional analysis to explain why the produced particles can strongly influence the dynamics of spacetime at the Planck time. The details are actually more involved than this, because one needs to remove the divergences in the energy-momentum tensor of the matter field before it can be used as the source of the Einstein equation to solve for the metric functions. This is what has been called the ‘backreaction’ problem. Zel'dovich and Starobinsky (1971) were the first to attempt such a calculation for conformal scalar fields in a Bianchi I universe. (See also Lukash and Starobinsky 1974). That was before the basic issues of particle creation processes were fully understood (e.g., ambiguities in the choice of the vacuum in curved space, see, Fulling 1973) and the regularization techniques were perfected. But they managed to show the viability of such processes. It took several years (1974-77) before the common ground between the different ways of regularization (e.g., adiabatic, point-splitting, dimensional, zeta-function) was reached and a firmer foundation for this new theory of semiclassical gravity was established. Hawking’s discovery (1974) of black hole radiance, and the confirmed ‘legality’ of trace anomaly (Capper and Duff 1975, Deser, Duff and Isham 1976) certainly invigorated this endeavour. The theoretical importance of the subject and the infusion of talents from general relativity, field theory, and particle physics have made this field an important component of contemporary theoretical physics (Birrell and Davies 1982). It is, as we shall see, also an intermediate step (the low energy or adiabatic limit) towards a

theory of quantum gravity and quantum cosmology.

Vigorous calculation of the backreaction of particle creation with regularized energy momentum sources began with the work of Hu and Parker (1977,1978) using adiabatic regularization methods, and Hartle (1977), Fischetti et al (1979) and Hartle and Hu (1979, 1980), using effective action methods. Their results on the effect of particle creation on anisotropy dissipation confirmed the qualitative estimate of Zel'dovich, and lent strong support for the viability of Misner's chaotic cosmology program.

1.2. Entropy of Quantum Fields and Spacetime?

These efforts in quantum field theory in curved spacetime and dissipative processes in the early universe contained the germs for two interesting directions of later development. Both involve quantum field theory and statistical mechanics applied to cosmological problems. They are

i) Dissipation in quantum fields as a statistical thermodynamic process:

Can one associate a viscosity with vacuum particle creation (Zel'dovich 1979, Hu 1984) like that of neutrino in the lepton era, or leptoquark boson decays in the GUT era? Can one define an entropy of quantum fields? (Hu and Pavon 1986, Hu and Kandrup 1987, Kandrup 1988)? An entropy of spacetime? (Penrose 1979, Hu 1983, Davies, Ford and Page 1986) Can one rephrase the chaotic cosmology idea in terms of some analog of second law of thermodynamics involving both fields and geometry (Hu 1983)? Can one generalize the fluctuation-dissipation relation (FDR) for quantum processes in spacetimes with event horizons (Candelas and Sciama 1977, Mottola 1986) to general dynamical spacetimes? Indeed can one view the backreaction process as a manifestation of a generalized FDR for dynamical systems like quantum fields in cosmological spacetimes (Hu 1989, Hu and Sinha 1993)?

This first direction of research has been pursued in the eighties, with the infusion of ideas and techniques in statistical and thermal field theory. It is intended to be a generalization of thermodynamic ideas successfully applied to black hole mechanics (Davies 1978) to non-equilibrium quantum field systems characteristic of cosmological problems. This was an offshoot of Misner's interest in dissipative processes in the early universe, only that one now takes on the challenge of doing everything with quantum fields in non-equilibrium conditions. This exercise probes into many deeper issues of thermodynamics and cosmology, such as the statistical nature of gravitational systems as different from ordinary matter, the thermodynamic nature of spacetimes (with or without event horizons), the nature of irreversibility and the cosmological origin of time-asymmetry. We will not belabor this direction of research here, but refer the reader to some recent reviews (Hu 1989, 1991, 1993a, 1993b). As we will see, inquiries into the statistical nature of particle creation and backreaction processes

actually have a role to play in the second direction, i.e.,

ii) Quantum field theory as the semiclassical limit of quantum gravity. How good is modified Einstein's theory with backreaction from quantum matter fields an approximation to quantum gravity? Under what conditions can the quantum dynamics of spacetime described by a wave function make the transition to the classical picture described by trajectories in superspace? We will use the paradigms of quantum open systems applied to Misner's minisuperspace models to illustrate these ideas.

1.3. From Dissipation to Quantum Open Systems

Speaking from the path traversed by one of us, one can identify a few critical nodes in the search for a pathway which connects i) particle creation as a dissipative processes to ii) backreaction and semiclassical gravity to iii) the statistical aspects in the issue of quantum to classical transition which underlies the relation of quantum gravity and semiclassical gravity. (See the Ph.D. thesis of Sinha 1991 for an overview of other major developments and references in these areas.) The (in-out) effective action formalism (Schwinger 1951, DeWitt 1964, 1975) was a correct start, because it gives the rate of particle production and the backreaction equations in a self-consistent manner (Hartle and Hu 1979). However the conditions for calculating the in-out vacuum persistence amplitude are unsuitable for the consideration of backreactions, as the effective equations are acausal and complex. The second major step was the recognition by DeWitt (1986), Jordan (1986), and Calzetta and Hu (1987) that the in-in or closed-time-path formalism of Schwinger (1961) and Keldysh (1964) is the right way to go. Calzetta and Hu (1987) derived the backreaction equations for conformal scalar fields in Bianchi I universe, which provided a beachhead for later discussion of the semiclassical limit of quantum cosmology. They also recognized the capacity of this framework in treating both the quantum and statistical aspects of dynamical fields and indeed used it for a reformulation of non-equilibrium statistical field theory (Calzetta and Hu 1988). The third step in making the connection with statistical mechanics is the construction of the so-called coarse-grained effective action by Hu and Zhang (1990), similar in spirit to the well-known projection operator method in statistical mechanics (Zwanzig 1961).

This is where the connection with 'quantum open systems' first enters. The general idea behind the study of such systems is that one starts with a closed system (universe), splits it into a "relevant" part (system) and an "irrelevant" (environment) part according to the specific physical conditions of the problem (for example, the split between light and heavy modes or slow and fast variables), and follows the behavior only of the relevant variables (now comprising an open system) by integrating out the irrelevant variables. (See Hu 1993a and 1993b for a discussion of the meaningfulness of such splits in the general physical and cosmological context

respectively). Thus the coarse-grained effective action generalizes the background field/ fluctuation-field splitting usually assumed when carrying out a quantum loop expansion in the ordinary effective action calculation to that between a system field and an environment field. It takes into account the averaged effect of the environment variables on the system and produces an effective equation of motion for the system variables. This method was used by Hu and Zhang (1990) (see also Hu 1991) to study the effect of coarse-graining the higher modes in stochastic inflation, and by Sinha and Hu (1991) to study the validity of minisuperspace approximations in quantum cosmology. The last step in completing this tour is the recognition by Hu, Paz and Zhang (1992) that the coarse-grained closed-time-path effective action formalism is equivalent to the Feynman-Vernon (1963) influence functional which was popularized by Caldeira and Leggett (1983) in their study of quantum macroscopic dissipative processes. Paz and Sinha (1991, 1992) have used this paradigm for a detailed analysis of the relation of quantum cosmology with semiclassical gravity. We like to illustrate this development with two problems, initially proposed and studied by Misner. One is on the validity of minisuperspace approximation, the other is on the classical limit of quantum cosmology.

2. Validity of the Minisuperspace Approximation

In quantum cosmology, the principal object of interest is the wave function of the universe $\Psi(h_{ij}, \phi)$, which is a functional on superspace. It obeys the Wheeler DeWitt equation, which is an infinite-dimensional partial differential equation. Its solution is a formidable task in the general case. To make the problem more tractable, Misner suggested looking at only a finite number of degrees of freedom (usually obtained by imposing a symmetry requirement), and coined the word minisuperspace (Misner 1972) quantization. We note that such a tremendous simplification has its preconditions. Since in the process of this transition one is truncating an infinite number of modes, nonlinear interactions of the modes among themselves and with the minisuperspace degrees of freedom are being ignored. It is also well-known that this truncation violates the uncertainty principle, since it implies setting the amplitudes and momenta of the inhomogeneous modes simultaneously to zero. It is therefore important to understand under what conditions it is reasonable to consider an autonomous evolution of the minisuperspace wave function ignoring the truncated degrees of freedom. Misner was keenly aware of the problems when he introduced this approximation. The first attempt to actually assess the validity of the minisuperspace approximation was made only recently by Kuchar and Ryan (1986, 1989).

Two of us (Sinha and Hu 1991) have tried to address this question in the context of interacting field theory with the help of the coarse-grained effective action method mentioned above. The model considered is that of a self interacting ($\lambda\phi^4$) scalar field coupled to a closed Robertson Walker background spacetime. The “system” (minisuperspace) consists of the scale factor a and the lowest mode of the scalar

field, while the “environment” consists of the inhomogeneous modes.¹ In this model calculation the scalar field is mimicking the inhomogeneous gravitational degrees of freedom in superspace. We are motivated to do this because linearized gravitational perturbations (gravitons) in a special gauge can be shown to be equivalent to a pair of minimally coupled scalar fields (Lifshitz 1946). Quantum cosmology of similar models of gravitational perturbations and scalar fields have been studied by several authors (Halliwell and Hawking 1985, Kiefer 1987, Vachaspati and Vilenkin 1988).

Our basic strategy will be to try to derive an “effective” Wheeler-DeWitt equation for the minisuperspace sector which contains the “averaged” effect of the higher modes as a backreaction term. We will then explicitly calculate the backreaction term using the effective action and consequently present a criterion for the validity of the minisuperspace approximation.

2.1. Effective Wheeler DeWitt Equation

The gravitational and matter actions in our model are given by

$$S_g = \frac{1}{2} \int d\eta a^2 (1 - \frac{a'^2}{a^2}) \quad (1)$$

$$S_m = -\frac{1}{2} \int \sqrt{-g} d^4x [\Phi \square \Phi + m^2 \Phi^2 + \frac{2\lambda}{4!} \Phi^4 + \frac{R}{6} \Phi^2] \quad (2)$$

where a is the scale factor of a closed Robertson-Walker universe, a factor $l_p^2 = 2/(3\pi m_p^2)$ is included in the metric for simplification of computations, and η is the conformal time. \square is the Laplace Beltrami operator on the metric $g_{\mu\nu}$, and m is the mass of the conformally coupled scalar field. Defining a conformally related field $\chi = (al_p)\Phi$ and expanding χ in scalar spherical harmonics $Q_{lm}^k(x)$ on the S^3 spatial sections,

$$\chi = \frac{\chi_0(t)}{(2\pi^2)^{\frac{1}{2}}} + \sum_{klm} f_{klm} Q_{lm}^k(x) \quad (3)$$

where $k = 2, 3, \dots, \infty, l = 0, 1, \dots, k-1, m = -l, -l+1, \dots, l-1, l$ (henceforth we will use k to denote the set $\{klm\}$). We will make the further assumption that the interactions of orders higher than quadratic of the lowest(minisuperspace) mode χ_0 with the higher modes f_n 's ($\sim \chi_0 f_k f_l f_m$) as well as the quartic self interaction of the higher modes ($\sim f_m f_n f_k f_l$) are small and can be neglected. With this assumption and with the following redefinitions $m^2 \rightarrow m^2/l_p^2, \lambda \rightarrow \lambda/2\pi^2$, the matter action can

¹In this example of minisuperspace the scalar field should not be thought of as providing a matter source for the Robertson-Walker background metric, since in that case varying the action with respect to the scale factor will not give the full set of Einstein equations. In particular, the $G_{ij} = 8\pi G T_{ij}$ equations that constrain the energy momentum tensor via $T_{ij} = 0$ for $i \neq j$ will be missing.

be written as

$$S_m = \int d\eta \left\{ \frac{1}{2} [\dot{\chi}_0^2 - m^2 a^2 \chi_0^2] - \frac{\lambda}{4!} \chi_0^4 - \frac{1}{2} \sum_k f_k \left[\frac{d^2}{d\eta^2} + k^2 \right] f_k - \frac{1}{2} \sum_k m^2 a^2 f_k^2 - \frac{\lambda}{4} \sum_k \chi_0^2 f_k^2 \right\} \quad (4)$$

The Hamiltonian constructed from the action $S = S_g + S_m$ is given by

$$H = -\frac{1}{2} \pi_a^2 + \frac{1}{2} \pi_{\chi_0}^2 + \frac{1}{2} \sum_n \pi_{f_n}^2 + V_0(a, \chi_0) + V(a, \chi_0, f_n) \quad (5)$$

where

$$V_0(a, \chi_0) = -\frac{1}{2} a^2 + \frac{1}{2} m^2 a^2 \chi_0^2 + \frac{\lambda}{4!} \chi_0^4 \quad (6)$$

and

$$V(a, \chi_0, f_n) = \frac{\lambda}{4} \sum_k \chi_0^2 f_k^2 + \frac{1}{2} \sum_k (k^2 + m^2 a^2) f_k^2 \quad (7)$$

where $\pi_a, \pi_{\chi_0}, \pi_{f_n}$ are the momenta canonically conjugate to a, χ_0, f_n respectively. The Wheeler-DeWitt equation for the wave function of the Universe Ψ is obtained from the Hamiltonian constraint by replacing the momenta by operators in the standard way, and is given by

$$\left[\frac{1}{2} \frac{\partial^2}{\partial a^2} - \frac{1}{2} \frac{\partial^2}{\partial \chi_0^2} - \frac{1}{2} \sum_n \frac{\partial^2}{\partial f_n^2} + V_0 + V \right] \Psi(a, \chi_0, f_n) = 0 \quad (8)$$

where we choose a factor ordering such that the kinetic term appears as the Laplace-Beltrami operator on superspace.

Writing

$$\Psi(a, \chi_0, f_n) = \Psi_0(a, \chi_0) \Pi_n \Psi_n(a, \chi_0, f_n) \quad (9)$$

we would like to obtain from (2.8) an effective Wheeler-DeWitt equation of the form

$$(H_0 + \Delta H) \Psi_0(a, \chi_0) = 0 \quad (10)$$

where H_0 is the part of the Hamiltonian operator in (2.8) independent of f_n and ΔH represents the influence of the higher modes. By making the assumption that Ψ_n varies slowly with the minisuperspace variables (see Sinha and Hu 1991 for details of this approximation) one can identify

$$\Delta H = - \sum_n \langle H_n \rangle \quad (11)$$

where the expectation value is taken with respect to Ψ_n . It is evident that the examination of this term will enable us to comment on the validity of the minisuperspace

description. We will then make a further assumption that Ψ_0 has a WKB form, i.e., $\Psi_0 = e^{iS(a, \chi_0)}$ which can be used in regions of superspace where the wavefunction oscillates rapidly, such that using eqn. (2.10), S satisfies

$$\frac{1}{2}(\nabla S)^2 + V_0 = - \sum_n \langle H_n \rangle \quad (12)$$

where ∇ is the gradient operator on minisuperspace. The above equation can be regarded as a Hamilton-Jacobi equation with backreaction. It can be also shown that the Ψ_n 's satisfy a Schrödinger - like equation with respect to the WKB time. This approximation can therefore be roughly thought of as the semiclassical limit where the minisuperspace modes behave classically, but the higher modes behave quantum mechanically (for further subtleties regarding the semiclassical limit see Sec. 3). Identifying $\frac{\partial S}{\partial a} = \pi_a$ and $\frac{\partial S}{\partial \chi_0} = \pi_{\chi_0}$, and substituting for the canonical momenta in terms of “velocities” a' and χ_0' Eq. (12) reduces to

$$\frac{1}{2}a'^2 - \frac{1}{2}\chi_0'^2 + V_0(a, \chi_0) = - \sum_n \langle H_n \rangle \quad (13)$$

This is the effective Wheeler- De Witt equation in the WKB limit, which we will compare with the backreaction equation derived in the next section, in order to calculate the term on the right hand side.

2.2. Backreaction of the Inhomogeneous Modes

We would now like to calculate the backreaction term (2.11) explicitly. Since we are making a split of the system from the environment based on the mode decomposition, we should use the coarse-grained effective action where the coarse graining consists of functionally integrating out the higher modes. As we would like to generate vacuum expectation values from the effective action rather than the matrix elements one should use the in-in or closed-time-path (CTP) version of the effective action (Calzetta and Hu 1987) rather than the in-out version. The CTP coarse-grained effective action in our case is given by (for details, see Sinha and Hu 1991)

$$e^{iS_{eff}(a^+, \chi_0^+, a^-, \chi_0^-)} = \int \mathcal{D}f_k^+ \mathcal{D}f_k^- e^{i(S(a^+, \chi_0^+, f_k^+) - S^*(a^-, \chi_0^-, f_k^-))} \quad (14)$$

S^* indicates that in this functional integral, m^2 carries an $i\epsilon$ term. $a^\pm, \chi_0^\pm, f_k^\pm$ are the fields in the positive (negative) time branch running from $\eta = \mp \infty$ to $\pm \infty$. The path integral is over field configurations that coincide at $t = \infty$. $\mathcal{D}f_k$ symbolizes the functional integration measure over the amplitudes of the higher modes of the scalar field.

We have derived the one loop renormalized coarse-grained effective action given as follows (omitting terms that involve $-$ fields only)

$$\begin{aligned}
 S_{eff} = & S_{g+} + \frac{1}{2} \int d\eta \left\{ \chi_0^{+'2} - \tilde{m}^{+2} \chi_0^{+2} \right\} \\
 & - \frac{\lambda}{4!} \int d\eta \chi_0^{+4} + \frac{13\lambda}{48} \int d\eta M^{+2} + \frac{1}{16} \int d\eta M^{+4} \\
 & + \frac{1}{32} \int d\eta_1 d\eta_2 M^{+2}(\eta_1) K(\eta_1 - \eta_2) M^{+2}(\eta_2) \\
 & + \frac{1}{32} \int d\eta_1 d\eta_2 M^{+2}(\eta_1) \bar{K}(\eta_1 - \eta_2) M^{-2}(\eta_2)
 \end{aligned} \tag{15}$$

where the coupling constants have their renormalized values. S_{g+} represents the classical gravitational part of the action. $M^{\pm 2} = \tilde{m}^{\pm 2} + \frac{1}{2}\lambda\chi_0^{\pm 2}$, and K and \bar{K} are complex nonlocal kernels with explicitly known forms. The effective equations of motion are obtained from this effective action via ²

$$\left. \frac{\delta S_{eff}}{\delta a^+} \right|_{\substack{a^+ = a^- = a \\ \chi_0^+ = \chi_0^- = \chi_0}} = 0 \quad \text{and} \quad \left. \frac{\delta S_{eff}}{\delta \chi_0^+} \right|_{\substack{a^+ = a^- = a \\ \chi_0^+ = \chi_0^- = \chi_0}} = 0 \tag{16}$$

Since we are interested in comparing with (2.13) which is equivalent to the G_{00} Einstein equation with backreaction, we need the first integral form of (2.16), which can be derived as

$$\begin{aligned}
 & \frac{1}{2}a'^2 - \frac{1}{2}\chi_0'^2 + \frac{1}{2}m^2a^2\chi_0^2 - \frac{1}{2}a^2 + \frac{\lambda}{4!}\chi_0^4 - \frac{13\lambda}{96}\chi_0^2 - \frac{13}{48}m^2a^2 - \frac{1}{16}M^2\ln\mu a \\
 & + \frac{1}{32} \int d\eta_1 M^2(\eta) \mathcal{K}(\eta - \eta_1) M^2(\eta_1) = 0
 \end{aligned} \tag{17}$$

with the assumption of having no quanta of the higher modes in the initial state. $\mathcal{K} = K + \bar{K}$ and is real and hence the above equation is also manifestly real. Equation (2.17) is then equivalent to the effective G_{00} Einstein equation or the Einstein- Hamilton- Jacobi equation plus backreaction. This in turn can be identified with (2.13), the effective Wheeler- De Witt equation in the WKB limit. Therefore, comparing (2.17) and (2.13) we can identify the backreaction piece in (2.13) as

$$\begin{aligned}
 \sum_n \langle H_n \rangle = & -\frac{13\lambda}{96}\chi_0^2 - \frac{13\lambda}{48}m^2a^2 - \frac{1}{16}M^2\ln\mu a \\
 & - \frac{1}{32} \int d\eta_1 M^2(\eta) \mathcal{K}(\eta - \eta_1) M^2(\eta_1)
 \end{aligned} \tag{18}$$

when the boundary conditions on the wave function are appropriate for the Ψ_n 's to be in a conformal “in” vacuum state. ³

²we need to compute only those terms in the effective action that involve the $+$ fields. Terms containing only $-$ fields will not contribute to the equations of motion.

³Since \mathcal{K} is real the backreaction term given above is real and represents a genuine expectation value in the “in” vacuum state rather than an in-out matrix element generated using the in-out effective action.

Since equation (2.17) is the “effective” Wheeler- DeWitt equation for the minisuperspace sector within our approximation scheme, the condition for validity of this approximation can be stated as

$$\sum_n \langle H_n \rangle \ll V_0 \quad (19)$$

where by the left hand side we mean the regularized value given by (2.18). It was shown that the term in equation (2.18) involving the nonlocal kernel is related to dissipative behavior in closely related models (Calzetta and Hu 1989). This dissipative behavior in turn has been related to particle production by the dynamical background geometry in semiclassical gravity models (Hu, 1989). In our case this can be interpreted as scalar particles in the higher modes being produced as a result of the dynamical evolution of the minisuperspace degrees of freedom generating a backreaction that modifies the minisuperspace evolution. We can therefore think of this term as introducing dissipation in the minisuperspace sector due to interaction with the higher modes that are integrated out. One can justifiably think of autonomous minisuperspace evolution only when this dissipation is small. Since we have used the scalar field modes to simulate higher gravitational modes these considerations can also be directly extended to include gravitons.⁴

This is an example of how ideas of open systems can be useful in understanding dissipation and backreaction, even in quantum cosmology. One can also use this paradigm to address the problem of quantum to classical transition, specifically the relation of semiclassical gravity with quantum gravity, which we will now address. To do this the formalism will need to be elevated to the level of density matrices.

3. Semiclassical Limit of Quantum Cosmology

We now report on the result of some recent work by two of us (Paz and Sinha 1991, 1992) on this problem. Quantum cosmology rests on the rather bold hypothesis that the entire universe can be described quantum mechanically. This pushes us to question the usual Copenhagen interpretational scheme that relies on the existence of an *a priori* classical external observer/apparatus. Since in the case of quantum cosmology this cannot be assumed, the theory needs to predict the “emergence” of a quasiclassical domain starting from a fundamentally quantum mechanical description. Recently, there has been a lot of interest in this subject, and two basic criteria for classicality have emerged (see Halliwell 1991 and references therein) from these endeavors. The first is *decoherence* – which requires that quantum interference between distinct alternatives must be suppressed. The second is that of *correlation*, which requires that the wave function or some distribution constructed from it (e.g. Wigner 1932) predicts

⁴A similar idea has been discussed by Padmanabhan and Singh (1990) in a linearized gravity model where they claim that in order that the minisuperspace approximation be valid, the rate of production of gravitons should be small.

correlations between coordinates and momenta in accordance with classical equations of motion. We will study the emergence of a semiclassical limit with the help of a quantum open system paradigm applied in the context of quantum cosmology. The basic mechanism for achieving decoherence and the appearance of correlations is by coarse-graining certain variables acting as an environment coupled to the system of interest.

3.1. Reduced Density Matrix

We will consider a D -dimensional minisuperspace with coordinates r^m ($m = 1, \dots, D$) as our system for which the Hamiltonian can be written as:

$$H_g = \frac{1}{2M} G^{mm'} p_m p_{m'} + MV(r^m) \quad (1)$$

where $G^{mm'}$ is the (super)metric of the minisuperspace and M is related to the square of the Planck mass. The minisuperspace modes are coupled to other degrees of freedom Φ , such as the modes of a scalar field, or the gravitational wave modes with a Hamiltonian $H_\Phi(\Phi, \pi_\Phi, r^m, p_m)$. These constitute the environment, or the ‘irrelevant’ part in our model. Thus, the wave function of the Universe is a function $\Psi = \Psi(r^m, \Phi)$ which satisfies the Wheeler-De Witt equation (Wheeler 1968, DeWitt 1967) :

$$H\Psi = (H_g + H_\Phi)\Psi = 0 \quad (2)$$

where the momenta are now replaced by operators and H_Φ . We use the same factor ordering as before. If one is interested in making predictions only about the behavior of the minisuperspace variables, a suitable quantity for such a coarse-grained description is the reduced density matrix defined as:

$$\rho_{red}(r_2, r_1) = \int d\Phi \Psi^*(r_1, \Phi) \Psi(r_2, \Phi) \quad (3)$$

We will consider an ansatz for the wave function of the following form:

$$\Psi(r, \Phi) = \sum_n e^{iMS_{(n)}(r)} C_{(n)}(r) \chi_{(n)}(r, \Phi). \quad (4)$$

where S_n is a solution of a Hamilton-Jacobi equation with respect to the r coordinate which is the same as (2.12) without the backreaction term and with the gradient given by $G^{mm'} \frac{\partial}{\partial r^{m'}}$. The prefactor $C_{(n)}$ is determined through the equation $2G^{mm'} \partial_m C_{(n)} \partial_{m'} S_{(n)} + \square_G S_0 = 0$, and $\chi_{(n)}$ is a solution of a Schrödinger equation with respect to the WKB time $\frac{d}{dt} = G^{mm'} \frac{\partial S_0}{\partial r^{m'}} \frac{\partial}{\partial r^m}$ and with Hamiltonian H_Φ . These equations are derived by the well-known order by order expansion in M^{-1} , where M acts as a parameter analogous to the mass of the heavy modes (in our case the minisuperspace ones) in the Born-Oppenheimer approximation. The Hamilton-Jacobi

equation satisfied by $S_{(n)}$ will have a $D-1$ parameter family of solutions, the subindex (n) labeling such parameters characterizes a particular solution and thus a specific WKB branch.

The reduced density matrix associated with the wave function (3.4) is then given by:

$$\rho_{red}(r_2, r_1) = \sum_{n, n'} e^{iM[S_{(n)}(r_1) - S_{(n')}(r_2)]} C_{(n)}(r_1) C_{(n')}(r_2) I_{n, n'}(r_2, r_1) \quad (5)$$

where we call

$$I_{n, n'}(r_2, r_1) = \int \chi_{(n')}^*(r_2, \Phi) \chi_{(n)}(r_1, \Phi) d\Phi \quad (6)$$

the influence functional. The justification for this name comes from the fact that it has been shown (Paz and Sinha 1991, Kiefer 1991) that it is exactly analogous to the Feynman-Vernon (1963) influence functional in the case in which the environment is initially in a pure state. For models with $r > 1$ it can be shown that it is indeed a functional of two histories, which is in turn related to the Gell-Mann Hartle decoherence functional (Gell-Mann and Hartle 1990, Griffith 1984, Omnes 1988). It is the central object of our consideration.

3.2. Decoherence and Correlation

We will now study the emergence of classical behavior in the minisuperspace variables as a consequence of their interaction with the environment. As we will show the two basic characteristics of classicality, the issues of correlations and decoherence, are indeed interrelated.

i) Decoherence: Decoherence occurs when there is no interference effect between alternative histories. If each of the diagonal terms in the sum of (3.4) corresponds to a nearly classical set of histories, the interference between them is contained in the non-diagonal terms ($n \neq n'$) of that sum. Thus, the system decoheres if influence functional $I_{n, n'} \propto \delta_{n, n'}$ approximately.⁵ As a consequence the density matrix (3.4) can be considered to describe a “mixture” of non-interfering WKB branches representing distinct Universes.

ii) Correlations: The second criterion for classical behavior is the existence of correlations between the coordinates and momenta which approximately obey the classical equations of motion. To analyze this aspect we will compute the Wigner function associated with the reduced density matrix and examine whether the Wigner function has a peak about a definite set of trajectories in phase space corresponding to the above correlations (Halliwell 1987, Padmandabhan and Singh 1990). Once we have established the decoherence between the WKB branches using criterion i), we

⁵for a suggestive argument to justify the choice of $I_{n, n'}(r, r)$ as an indicator of the degree of decoherence between the WKB branches, see Paz and Sinha (1992) Sec. V

look for correlations using the Wigner function *within* a “decohered WKB branch”, i.e., that associated with ($n = n'$). In this sense the two criteria are interrelated, i.e., we *need* decoherence between the distinct WKB branches to be able to meaningfully predict correlations.

The Wigner function associated with one of the diagonal terms is given by

$$W_{(n)}(r, P) = \int_{-\infty}^{+\infty} d\xi^m C_{(n)}(r_1) C_{(n)}(r_2) e^{-2iP_m \xi^m} e^{iM[S_{(n)}(r_1) - S_{(n)}(r_2)]} I_{n,n}(r_2, r_1) \quad (7)$$

where $r_{1,2}^m = r^m \pm \frac{\xi^m}{M}$.

The functional $I_{n,n}(r_2, r_1)$ plays the dual role of producing the diagonalization of $\rho_{(n)}$ and affecting the correlations (the phase affects the correlations and the absolute value the diagonalization).⁶ In the language of measurement theory (Wheeler and Zurek 1986) one can say that the environment is “continuously measuring” the minisuperspace variables (Zurek 1981, 1991, Zeh 1986, Kiefer 1987) and that this interaction not only suppresses the $n \neq n'$ terms in the sum of equation (3.11), but also generates a “localization” of the r variables inside each WKB branch. This localization effect is essential in order to obtain a peak in the Wigner function. The form of the peak and the precise location of its center are determined by the form of $I_{n,n}(r_2, r_1)$. To illustrate this better let us assume that the state of the environment is such that the influence functional can be written in the form

$$I_{(n,n)}(r) \simeq e^{i\beta_m(r)\xi^m - \sigma^2 \xi_m \xi^m} \quad (8)$$

where β and σ are real and $r = \frac{r_1 + r_2}{2}$, and we notice that σ is related to the degree of diagonalization of the density matrix. The Wigner function for this is computed to be

$$W_{(n)}(r, P) \simeq C_{(n)}^2(r) \sqrt{\frac{\pi}{\sigma^2}} e^{-(P_m - M \frac{\partial S}{\partial r^m} - \frac{1}{2}\beta_m)^2} \quad (9)$$

It is a Gaussian peaked about the classical trajectory ($P_m - M \frac{\partial S}{\partial r^m} = 0$) shifted by a backreaction term $\frac{1}{2}\beta_m$, but with a spread characterized by σ , both of these arising from coarse graining the environment. We also notice a competition between sharp correlations (related to the sharpness of the above peak) and the diagonalization of the density matrix, and this can be formalized (see Paz and Sinha 1991, 1992) in a set of criteria for the emergence of classical behavior through a compromise between decoherence and correlations. It can also be shown that when the influence

⁶The diagonalization produced by $I_{n,n}(r_2, r_1)$ has been studied by various authors (Kiefer 1987, Halliwell 1989, Padmandabhan 1989, Laflamme and Louko 1991) and has been identified with decoherence. However, as noted by Paz and Sinha (1992), this term applies more properly to the lack of interference between WKB branches which, as emphasized by Gell-Mann and Hartle (1990), is the more relevant effect. However, it is clear that the diagonalization is an effect that accompanies the former one and has the same origin.

functional is of the form (3.8) the correlations predicted are exactly those given by the semiclassical Einstein equations. Thus the above arguments can be tied to the emergence of the semiclassical limit of quantum fields in curved space time starting from quantum cosmology. This has been illustrated in Paz and Sinha (1991, 1992) in the context of various specific cosmological models.

4. Statistical Mechanics and Quantum Cosmology

The above two examples give a good illustration of how models of quantum open systems can be used to understand some basic issues of quantum cosmology. The advantage of using statistical mechanical concepts for the study of issues in quantum cosmology and semiclassical gravity has been discussed in general terms by Hu (1991) who emphasized the importance of the interconnection between processes of decoherence, correlation, dissipation, noise and fluctuation, particle creation and backreaction. (see also Hu, Paz and Zhang 1992, 1993, and Gell-Mann and Hartle 1993). In our view, these processes are actually different manifestations of the effect of the environment on the different attributes of the system (the phase information, energy distribution, entropy content, etc). Decoherence is related to particle creation as they both can be related to the Bogolubov coefficients, as is apparent in the example of Calzetta and Mazzitelli (1991) and Paz and Sinha (1992). It is also important to consider the nature of noise and fluctuation for any given environment. In this way their overall effect on the system can be captured more succinctly and effectively by general categorical relations like the fluctuation-dissipation relations (see Hu 1989, Hu and Sinha 1993). This also sheds light on how to address questions like defining gravitational entropy and related questions mentioned in the Introduction (see Hu 1993b).

So far one has only succeeded in finding a pathway to show how semiclassical gravity can be deduced from quantum cosmology. One important approximation which makes this transition possible is the assumption of a WKB wave function. To see the true colors of quantum gravity, which is nonlinear and is likely to be also nonlocal, one needs to avoid such simplifications. One should incorporate dynamical fluctuations both in the fields and in the geometry without any background field separation, and deal with nonadiabatic and nonlinear conditions directly. This is a difficult but necessary task. Calzetta (1991) has tackled the anisotropy dissipation problem in quantum cosmology without such an approximation. Recently Calzetta and Hu (1993) have proposed an alternative approach to address the quantum to classical transition issue in terms of correlations between histories. It uses the BBGKY hierarchy truncation scheme to provide a more natural coarse-graining measure which brings about the decoherence of correlation histories. This scheme goes beyond a simple system-environment separation and enables one to deal with nonadiabaticity and nonlinearity directly as in quantum kinetic theory.

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References

- A. Anderson, Phys. Rev. **D42**, 585 (1990)
- R. Arnowitt, S. Deser, and C. W. Misner, "The Dynamics of General Relativity." in *Gravitation: An Introduction to Current Research*, ed. L. Witten, pp. 227-65 (Wiley, New York, 1962)
- V. A. Belinskii, E. M. Lifschitz and I. M. Khalatnikov, Adv. Phys. **19**, 525 (1970)
- B. K. Berger, Phys. Rev. **D11**, 2770 (1975)
- B. K. Berger, Ann. Phys. (N. Y.) **83**, 203 (1974)
- L. Bianchi, *Mem. di. Mat. Soc. Ital. Sci.*, **11**, 267 (1897)
- N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, (Cambridge University Press, Cambridge, 1982).
- A. O. Caldeira and A. J. Leggett, Physica **121A**, 587 (1983)
- E. Calzetta, Class. Quant. Grav. **6**, L227 (1989)
- E. Calzetta, Phys. Rev. **D43**, 2498 (1991)
- E. Calzetta and B. L. Hu, Phys. Rev. **D35**, 495 (1987)
- E. Calzetta and B. L. Hu, Phys. Rev. **D37**, 2838 (1988)
- E. Calzetta and B. L. Hu, Phys. Rev. **D40**, 380 (1989)
- E. Calzetta and B. L. Hu, Phys. Rev. **D40**, 656 (1989)
- E. Calzetta and B. L. Hu, "Decoherence of Correlation Histories" in *Directions in General Relativity* Vol 2 (Brill Festschrift), eds. B. L. Hu, M. P. Ryan and C. V. Vishveshwara (Cambridge Univ. Cambridge, 1993)
- E. Calzetta and F. Mazzitelli, Phys. Rev. **D42**, 4066 (1991)
-)

- P. Candelas and D. W. Sciama, Phys. Rev. Lett. **38**, 1372 (1977)
- D. M. Capper and M. J. Duff, Nuovo Cimento **A23**, 173 (1975)
- S. Deser, M. J. Duff and C. J. Isham, Nucl. Phys. **B111**, 45 (1976)
- P. C. W. Davies, Rep. Prog. Phys. **41**, 1313 (1978)
- P. C. W. Davies, L. H. Ford and D. N. Page Phys. Rev. **D34**, 1700 (1986)
- B. S. DeWitt, Phys. Rev. **160**, 1113 (1967)
- B. S. DeWitt, Phys. Rep. **19C**, 297 (1975)
- B. S. DeWitt, in *Quantum Concepts in Space and Time*, ed. R. Penrose and C. J. Isham (Claredon Press, Oxford, 1986)
- H. F. Dowker and J. J. Halliwell, Phys. Rev. **D46**, 1580 (1992)
- D. Eardley, E. Liang and R. K. Sachs, J. Math. Phys. **13**, 99 (1971)
- G. F. R. Ellis and M. A. H. MacCallum, Commun. Math. Phys. **12**, 108 (1969)
- R. P. Feynman and F. L. Vernon, Ann. Phys. **24**, 118 (1963)
- M. V. Fischetti, J. B. Hartle and B. L. Hu, Phys. Rev. **D20**, 1757 (1979)
- S. A. Fulling, Phys. Rev. **D7**, 2850 (1973)
- M. Gell-Mann and J. B. Hartle, in *Complexity, Entropy and the Physics of Information*, ed. W. Zurek, Vol. IX (Addison-Wesley, Reading, 1990)
- M. Gell-Mann and J. B. Hartle, Phys. Rev. **D47** (1993)
(1988)
- R. Griffiths, J. Stat. Phys. **36**, 219 (1984)
- A. H. Guth, Phys. Rev. **D23**, 347 (1991)
- S. Habib, Phys. Rev. **D42**, 2566 (1990)
- S. Habib and R. Laflamme, Phys. Rev. **D42**, 4056 (1990)
- J. J. Halliwell, Phys. Rev. **D36**, 3627 (1987)
- J. J. Halliwell, Phys. Rev. **D39**, 2912 (1989)
ys.**A** (1990)
- J. J. Halliwell, Lectures at the 1989 Jerusalem Winter School, in *Quantum Mechanics and Baby Universes*, eds. S. Coleman, J. Hartle, T. Piran and S. Weinberg (World Scientific, Singapore, 1991)
- J. J. Halliwell and S. W. Hawking, Phys. Rev. **D31**, 1777 (1985)

- J. B. Hartle, Phys. Rev. Lett. **39**, 1373 (1977)
- J. B. Hartle, in *Gravitation in Astrophysics*, NATO Advanced Summer Institute, Cargese, 1986 ed. B. Carter and J. Hartle (NATO ASI Series B: Physics Vol. 156, Plenum, N.Y 1987).
- J. B. Hartle and S. W. Hawking, Phys. Rev. **D28**, 1960 (1983)
- J. B. Hartle and B. L. Hu, Phys. Rev. **D20**, 1772 (1979)
- J. B. Hartle and B. L. Hu, Phys. Rev. **D21**, 2756 (1980)
- S. W. Hawking, *Nature* (London), **248**, 30 (1974)
- O. Heckman and E. L. Schucking, In *Gravitation: An Introduction to Current Research*, ed. L. Witten (Wiley, New York, 1962)
- B. L. Hu, Phys. Lett. **A90**, 375 (1982)
- B. L. Hu, Phys. Lett. **A97**, 368 (1983)
- B. L. Hu, in *Cosmology of the Early Universe*, ed. L. Z. Fang and R. Ruffini (World Scientific, Singapore, 1984)
- B. L. Hu, Physica **A158**, 399 (1989)
- B. L. Hu "Quantum and Statistical Effects in Superspace Cosmology" in *Quantum Mechanics in Curved Spacetime*, ed. J. Audretsch and V. de Sabbata (Plenum, London 1990)
- B. L. Hu, in *Relativity and Gravitation: Classical and Quantum*, Proc. SILARG VII, Cocoyoc, Mexico 1990, eds. J. C. D' Olivo et al (World Scientific, Singapore 1991)
- B. L. Hu, "Statistical Mechanics and Quantum Cosmology", in *Proc. Second International Workshop on Thermal Fields and Their Applications*, eds. H. Ezawa et al (North-Holland, Amsterdam, 1991)
- B. L. Hu, "Fluctuation, Dissipation and Irreversibility" in *The Physical Origin of Time-Asymmetry*, Huelva, Spain, 1991, eds. J. J. Halliwell, J. Perez-Mercader and W. H. Zurek (Cambridge University Press, 1993)
- B. L. Hu, "Quantum Statistical Processes in the Early Universe" in *Quantum Physics and the Universe*, Proc. Waseda Conference, Aug. 1992 ed. Namiki, K. Maeda, et al (Pergamon Press, Tokyo, 1993)
- B. L. Hu and H. E. Kandrup, Phys. Rev. **D35**, 1776 (1987)
- B. L. Hu and L. Parker, Phys. Lett. **63A**, 217 (1977)
- B. L. Hu and L. Parker, Phys. Rev. **D17**, 933 (1978)
- B. L. Hu and D. Pavon, Phys. Lett. **B180**, 329 (1986)

- B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. **D45**, 2843 (1992)
- B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. **D47** (1993)
- B. L. Hu, J. P. Paz and Y. Zhang, "Stochastic Dynamics of Interacting Quantum Fields" Phys. Rev. D (1993)
- B. L. Hu and S. Sinha, "Fluctuation-Dissipation Relation in Cosmology" Univ. Maryland preprint (1993)
- B. L. Hu and Y. Zhang, "Coarse-Graining, Scaling, and Inflation" Univ. Maryland Preprint 90-186 (1990)
- E. Joos and H. D. Zeh, Z. Phys. **B59**, 223 (1985)
- R. D. Jordan, Phys. Rev. **D33**, 44 (1986)
- H. E. Kandrup, Phys. Rev. **D37**, 3505 (1988)
- L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JEPT **20**, 1018 (1965)]
- C. Kiefer, Class. Quant. Grav. **4**, 1369 (1987)
- C. Kiefer, Class. Quant. Grav. **8**, 379 (1991)
- K. Kuchar, Phys. Rev. **D4**, 955 (1971)
- K. Kuchar and M. P. Ryan Jr., in *Proc. of Yamada Conference XIV* ed. H. Sato and T. Nakamura (World Scientific, 1986)
- K. Kuchar and M. P. Ryan Jr., Phys. Rev. **D40**, 3982 (1989)
- R. Laflamme and J. Luoko, Phys. Rev. **D43**, 3317(1991)
- and W. Zwerger, Rev. Mod. Phys. 59 No. 1 (1987) 1
- E. M. Lifschitz and I. M. Khalatnikov, Adv. Phys. **12**, 185 (1963)
- V. N. Lukash and A. A. Starobinsky, JETP **39**, 742 (1974)
- R. A. Matzner and C. W. Misner, Ap. J. **171**, 415 (1972)
- C. W. Misner, Ap. J. **151**, 431 (1968)
- C. W. Misner, Phys. Rev. Lett. **22**, 1071 (1969)
- C. W. Misner, Phys. Rev. **186**, 1319 (1969)
- C. W. Misner, in *Magic Without Magic*, ed. J. Klauder (Freeman, San Francisco, 1972)
- E. Mottola, Phys. Rev. **D33**, 2136 (1986)

- R. Omnes, J. Stat. Phys. **53**, 893, 933, 957 (1988); Ann. Phys. (NY) **201**, 354 (1990); Rev. Mod. Phys. **64**, 339 (1992)
- T. Padmanabhan, Phys. Rev. **D39**, 2924 (1989)
- T. Padmanabhan and T. P. Singh, Class. Quan. Grav. **7**, 411 (1990)
- L. Parker, “The Creation of Particles in an Expanding Universe,” Ph.D. Thesis, Harvard University (unpublished, 1966).
- L. Parker, Phys. Rev. **183**, 1057 (1969)
- L. Parker, Phys. Rev. **D7**, 976 (1973)
- J. P. Paz, Phys. Rev. **D41**, 1054 (1990)
- J. P. Paz, Phys. Rev. **D42**, 529 (1990)
- .
- J. P. Paz and S. Sinha, Phys. Rev. **D44**, 1038 (1991)
- J. P. Paz and S. Sinha, Phys. Rev. **D45**, 2823 (1992)
- J. P. Paz and W. H. Zurek, in preparation (1993)
- R. Penrose, “Singularities and Time-Asymmetry” in *General Relativity: an Einstein Centenary Survey*, eds. S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979)
- M. P. Ryan, *Hamiltonian Cosmology* (Springer, Berlin, 1972)
- M. P. Ryan, Jr. and L. C. Shepley, *Homogeneous Relativistic Cosmologies* (Princeton University Press, Princeton, 1975)
- J. Schwinger, Phys. Rev., **82**, 664 (1951)
- J. S. Schwinger, J. Math. Phys. **2** 407 (1961).
- R. U. Sexl and H. K. Urbantke, Phys. Rev., **179**, 1247 (1969)
- S. Sinha, Ph. D. Thesis, University of Maryland (unpublished 1991)
- S. Sinha and B. L. Hu, Phys. Rev. **D44**, 1028 (1991).
- A. H. Taub, Ann. Math., **53**, 472 (1951)
- T. Vachaspati and A. Vilenkin, Phys. Rev. **D37**, 898 (1988)
- A. Vilenkin, Phys. Rev. **D27**, 2848 (1983), **D30**, 509 (1984); Phys. Lett. **117B**, 25 (1985)
- J. A. Wheeler, in *Relativity, Groups and Topology*, ed. B. S. De Witt and C. De Witt (Gordon and Breach, New York, 1964)

- J. A. Wheeler, in *Battelle Rencontres* ed. C. DeWitt and J.A Wheeler (Benjamin, N. Y. 1968)
- J. A. Wheeler and W. H. Zurek, *Quantum Theory and Measurement* (Princeton University Press, Princeton 1986)
- E. P. Wigner, Phys. Rev. **40**, 749 (1932)
- H. D. Zeh, Phys. Lett. **A116**, 9 (1986).
- Ya. B. Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz, **12** ,443 (1970) [JETP Lett. **12**, 307(1970)]
- Ya. B. Zel'dovich, in *Physics of the Expanding Universe*, ed. M. Diemianski (Springer, Berlin, 1979)
- Ya. B. Zel'dovich and A. A. Starobinsky, Zh. Teor. Eksp. Fiz. **61** (1971) 2161 [JEPT **34**, 1159 (1972)]
- W. H. Zurek, Phys. Rev. **D24**, 1516 (1981); **D26**, 1862 (1982); in *Frontiers of Nonequilibrium Statistical Physics*, ed. G. T. Moore and M. O. Scully (Plenum, N. Y., 1986); Physics Today **44**, 36 (1991)
- R. Zwanzig, in *Lectures in Theoretical Physics*, ed. W. E. Britten, B. W. Downes and J. Downes (Interscience, New York, 1961)